CBCS SCHEME

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21MAT21

Second Semester B.E. Degree Examination, Dec.2023/Jan.2024
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Evaluate : $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. **(06 Marks)**

b. Evaluate $\int_0^{\infty}\int_0^{x} xe^{-x^2/y}\,dy\,dx$ by changing the order of integration. **(07 Marks)**

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$. **(07 Marks)**

OR

2 a. Evaluate $\iint_R xy\,dx\,dy$ over the region R bounded by the x-axis, ordinate x = 2a and the curve $x^2 = 4ay$. **(06 Marks)**

b. Find the area of the ellipse using double integration $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **(07 Marks)**

c. Derive the relation between Gamma and Beta functions $\beta(m.n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$. **(07 Marks)**

Module-2

3 a. Find the directional derivative of $\phi = \frac{xz}{x^2+y^2}$ at the point (1, -1, 1) along the direction $\hat{i} - 2\hat{j} + \hat{k}$. **(06 Marks)**

b. Find the div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1) where $\vec{F} = \nabla(xy^3z^2)$. **(07 Marks)**

c. Find the constants a, b and c such that $\vec{F} = (axy - z^3)\hat{i} + (bx^2 + z)\hat{j} + (bxz^2 + cy)\hat{k}$ is irrotational. **(07 Marks)**

OR

4 a. Find the work done in moving a particle in the force field $\vec{F} = (2y - x^2)\hat{i} + 6yz\hat{j} - 8xz^2\hat{k}$ along the straight line from (0, 0, 0) to the point (1, 1, 1). **(06 Marks)**

b. Using the Green's theorem, evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$, where 'c' is the triangle formed by the lines x = 0, y = 0 and y + x = 1. **(07 Marks)**

c. Using Stoke's theorem, evaluate $\int_s (\text{curl}\vec{f}).\hat{n}ds$ for $\vec{f} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$, where s is the surface of the cube formed by the planes x = 0, y = 0, x = 2, y = 2 and z = 2 with its bottom removed. **(07 Marks)**

Module-3

5 a. Form a partial differential equation by eliminating arbitrary constants from $(x-a)^2 + (y-b)^2 = z^2$. **(06 Marks)**

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x-y) = 0$. **(07 Marks)**

c. With usual notation derive a one-dimensional wave equation. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating arbitrary functions from $\phi(xy+z^2, x+y+z) = 0$. **(07 Marks)**

b. Solve : $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$. **(07 Marks)**

c. Solve : $\frac{\partial^2 z}{\partial y^2} = z$, given that when y = 0, z = 0 and $\frac{\partial z}{\partial y} = \sin x$. **(06 Marks)**

Module-4

7 a. Using Regula-Falsi method, compute the real root which lies between 0.5 and 1 of the equation cosx = 3x-1, correct to three decimal places. **(06 Marks)**

b. Find the number of students who obtained marks between 40 and 45 from the following data:

Marks:	30-40	40-50	50-60	60-70	70-80
Number of students:	31	42	51	35	31

(07 Marks)

c. Evaluate $\int_4^{5.2} \log_e^x \, dx$, using the Simpson's 1/3rd rule, by dividing the interval into 6 equal parts. **(07 Marks)**

OR

8 a. By using Newton's – Raphson method find the real root of the equation x sinx + cosx = 0 near to x = π, correct to 3 decimal places (x is in radians). **(06 Marks)**

b. Using Newton's divided difference formula, find an interpolating polynomial which passes through the points (4, -43), (7, 83), (9, 327) and (12, 1058). **(07 Marks)**

c. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using the Simpson's 3/8th rule, dividing the interval into six equal parts and hence deduce the value of π. **(07 Marks)**

Module-5

9 a. Using Taylor's series method find the solution of $\frac{du}{dx} = x^2 + y^2$, with y(0) = 1 at x = 0.1 and x = 0.2 of order four. **(06 Marks)**

b. Solve the initial value problem $\frac{dy}{dx} = x + y^2$; with y(0) = 1 at x = 0.1 by taking h = 0.1 using the Runge-Kutta method of order 4. **(07 Marks)**

c. Find the value y at x = 1.4 using Milne's predictor – corrector method given that, $\frac{dy}{dx} = x^2 + \frac{y}{z}$ with y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649 and y(1.3) = 2.7514. Apply corrector formula twice. **(07 Marks)**

OR

10 a. Using modified Euler's method, solve the initial value problem $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$; with y(20) = 5 at x = 20.2 by taking h = 0.2 apply modification three times. **(06 Marks)**

b. Find the value of y at x = 0.1 given that $\frac{dy}{dx} = 3x + \frac{y}{z}$; y(0) = 1 by using Runge-Kutta method of order 4. (Take h = 0.1). **(07 Marks)**

c. Find y(1.4) using Milne's predictor-corrector method given that $\frac{dy}{dx} = x^2(1+y)$; with y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548 and y(1.3) = 1.979 apply corrector formula twice. **(07 Marks)**

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